

# Flavor Physics in $SO(10)$ GUTs with Suppressed Proton decay Due to Gauged Discrete Symmetry

A. T. Azatov and R. N. Mohapatra

*Maryland Center for Fundamental Physics and Department of Physics,  
University of Maryland,  
College Park, MD 20742, USA*

(Dated: February, 2008)

## Abstract

Generic  $SO(10)$  GUT models suffer from the problem that Planck scale induced non-renormalizable proton decay operators require extreme suppression of their couplings to be compatible with present experimental upper limits. One way to resolve this problem is to supplement  $SO(10)$  by simple gauged discrete symmetries which can also simultaneously suppress the renormalizable R-parity violating ones when they occur and make the theory “more natural”. Here we discuss the phenomenological viability of such models. We first show that for both classes of models, e.g the ones that use  $\mathbf{16}_H$  or  $\mathbf{126}_H$  to break B-L symmetry, the minimal Higgs content which is sufficient for proton decay suppression is inadequate for explaining fermion masses despite the presence of all apparently needed couplings. We then present an extended  $\mathbf{16}_H$  model, with three  $\mathbf{10}$  and three  $\mathbf{45}$ -Higgs, where is free of this problem. We propose this as a realistic and “natural” model for fermion unification and discuss the phenomenology of this model e.g. its predictions for neutrino mixings and lepton flavor violation.

## I. INTRODUCTION

The neutrino observations of the past decade have put the spotlight on gauged B-L symmetry as well as unification groups such as  $SO(10)$  and  $SU(2)_L \times SU(2)_R \times SU(4)_4$  containing B-L as prime candidates for theory of matter, forces and flavor. While both these groups incorporate the seesaw mechanism for neutrino masses,  $SO(10)$  has the additional attractive feature that gauge couplings unify at high scale. It is however highly nontrivial to obtain a “truly natural”  $SO(10)$  model due to such issues as doublet triplet splitting, rapid proton decay etc. In this paper we discuss how one aspect of this naturalness can be addressed i.e. how one can naturally suppress proton decay in  $SO(10)$  models while preserving our understanding neutrino masses.

We first note that  $SO(10)$  models for neutrinos discussed in recent literature can by and large be divided into two classes:

(i) One class which uses only renormalizable couplings involving the Higgs fields **10**, **120** and **126** for fermion masses and the last multiplet for breaking B-L symmetry and multiplets such as **45** and/or **210** for gauge symmetry breaking[1]. This theory could be considered as an ultraviolet complete theory by itself.

(ii) The second class uses **10** plus  $\mathbf{16} \oplus \bar{\mathbf{16}}$  for fermion masses with the **16**’s breaking the B-L symmetry. Here one generally uses **45**+**54** Higgs fields for  $SO(10)$  breaking. An important feature of this class is that it has to rely on nonrenormalizable couplings to understand fermion masses and therefore has to be viewed necessarily as an effective theory at the GUT scale[2].

The first class of models leads to automatic R-parity conservation when  $SO(10)$  breaks down to MSSM so that there is a natural candidate for dark matter whereas the second class of models suffers from R-parity breaking and hence has no stable dark matter in the absence of additional symmetries. So in principle one could argue that this class of models are not “pure”  $SO(10)$  models.

Both models have an additional naturalness problem arising from the fact that they allow R-parity conserving nonrenormalizable couplings of the form  $\lambda \mathbf{16}_m^4 / M_{Pl}$  which lead to rapid proton decay. Such interactions could be induced by nonperturbative Planck scale effects and it is therefore not safe to ignore them. Present proton life time limits constrain  $\lambda$  to be  $\leq 10^{-7}$ . Such a small value of  $\lambda$  would suggest that there is probably a symmetry responsible for its smallness. This question is particularly urgent for the class of  $SO(10)$  models with **16** Higgs since they rely on other such dimension four higher dimensional operators with coefficients of order one to un-

derstand fermion masses. This problem is generic to all non-GUT susy theories such as MSSM or left-right models as well as  $SU(2)_L \times SU(2)_R \times SU(4)_4$  models and not just GUT theories. One way to understand the suppression of such operators despite the presence of non-perturbative gravitational effects, is to have an additional gauge symmetry beyond  $SO(10)$  which can forbid these unwanted terms. The simplest possibility is to have a discrete gauge symmetry[3]. There are of course other possibilities[4].

The discrete gauge symmetry supplemented  $SO(10)$  models that suppress proton decay were studied for a large class of models in a recent paper[5]. In particular two minimal  $SO(10)$  models—one with **16**-Higgs breaking the B-L symmetry and another with **126** breaking B-L were shown to be free of both proton decay problem as well as R-parity problem if  $SO(10)$  was supplemented by a gauged  $Z_6$  symmetry. They looked promising for phenomenology since all necessary terms in the superpotential for phenomenology were allowed by the symmetry. It is the goal of this paper to study the viability of these models.

The results of this paper are the following: (i) the minimal versions of both **16**<sub>H</sub>-based as well as **126**-based models discussed in Ref.[5] are not realistic since they fail to give desired MSSM doublets that would be required to give rise to realistic fermion masses and mixings; (ii) if the **16**-based models are extended to have three **10**-Higgs fields and three **45** multiplets, one can have the desired doublet-triplet splitting and fermion masses that can match observations. This model differs from other **16**-based models in that proton decay here arises only from the gauge boson exchanges unlike other models where Planck scale induced effects as well as Higgsino exchange ones play a role[6]; (iii) we study the phenomenological implications of this model and isolate some of its tests e.g. in the domain of lepton flavor violation.

This paper is organized as follows: in sec. 2, we review the salient features of the two classes of models; in sec. 3, we discuss doublet-triplet splitting problem of the minimal models; in sec.4 we discuss the three Higgs extension of the **16**-based model that fits fermion masses and mixings; in sec. 5, we discuss how large neutrino mixings and observed neutrino masses arise in this model. We summarize our results in sec. 6.

## II. THE $SO(10) \times Z_6$ MODEL FOR 16-HIGGS B-L BREAKING

The main features of generic  $SO(10)$  models with **16**-Higgs fields breaking B-L symmetry are the following: (i) the quarks and leptons are assigned to three **16**-dimensional spinors (denoted by

$\psi_m$ ,  $m=1,2,3$ ); (ii) the GUT symmetry is broken down to  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$  by a  $\mathbf{45} \oplus \mathbf{54}$  set of Higgs fields; (iii)  $SU(2)_R \times U(1)_{B-L}$  symmetry is broken by the **16**-Higgs pair denoted by  $\psi_H \oplus \bar{\psi}_H$  set to the standard model symmetry which is then broken by  $SU(2)$  doublets that are linear combinations of the doublets in SO(10) **10** and **16**-Higgs fields. The standard model symmetry along with supersymmetry emerge just below the GUT scale of  $2 \times 10^{16}$  GeV.

This model has several naturalness problems: it not only allows the dangerous R-parity conserving  $\frac{(16_m)^4}{M_{Pl}}$  terms but also terms such as  $\frac{(16_m)^3 16_H}{M_{Pl}}$  terms which on B-L symmetry breaking lead to all three types of R-parity violating operators present in general MSSM i.e.  $LLe^c$ ,  $QLd^c$  and  $u^c d^c d^c$  type. Thus this model for natural values of couplings will lead to extremely rapid proton decay which is unacceptable. The question addressed in Ref.[5] is to search for gauged discrete symmetries that will keep the model phenomenologically viable while keeping them “proton decay safe” and it was shown that the minimal anomaly free discrete gauge symmetry is  $Z_6$ . Similar considerations for **126** type models also led to the symmetry  $Z_6$  and in both cases an extra **10**-Higgs field denoted by  $H'$  in addition to those considered already.

To see the discrete symmetry charges for various fields that forbid both R-parity violating terms as well as R-parity conserving baryon number violating terms, while at the same time keeping the required terms responsible for good phenomenology, we divide the superpotential terms into two classes: type I terms that must be kept for phenomenology and type II terms that must be forbidden to suppress proton decay and R-parity violating terms. They are given below:

**Terms of type I:** They include  $\psi_m \psi_m H$ ,  $(\psi_m \bar{\psi}_H)^2/M_P$ ,  $\psi_H \bar{\psi}_H$ ,  $A^2$ ,  $S^{2,3}$  and  $S A^2$ , where  $H$ ,  $A$ ,  $S$  are **10**-, **45**-, **54**-plets, respectively. Taking the discrete gauge symmetry to be  $Z_N$ , we can write down the constraints on the  $Z_N$  charges that are required by the type I terms:

$$2q_{\psi_m} + q_H = 0 \pmod{N}, \quad 2q_{\psi_m} + 2q_{\bar{\psi}_H} = 0 \pmod{N}, \quad (1a)$$

$$q_{\psi_H} + q_{\bar{\psi}_H} = 0 \pmod{N}, \quad q_H + q_{H'} = 0 \pmod{N}, \quad (1b)$$

$$2q_A = 0 \pmod{N}, \quad 2q_S = 0 \pmod{N}, \quad (1c)$$

$$3q_S = 0 \pmod{N}, \quad 2q_A + q_S = 0 \pmod{N}. \quad (1d)$$

Here, we denote the  $Z_N$  charge for a field  $F$  by  $q_F$ .

**Type II terms:** These are the terms that must be forbidden from appearing in the superpotential and are  $\psi_m \psi_m H'$ ,  $\psi_m^4$ ,  $\psi_m \bar{\psi}_H$  and  $\psi_m^3 \psi_H$ ,  $\psi_m \psi_H H$ ,  $\psi_m \psi_H H'$  and  $\psi_m \bar{\psi}_H A$ . We forbid the  $\psi_m \psi_m H'$  in order to avoid large Higgsino mediated contribution to proton decay since this is the very problem we are trying to solve. The necessary constraints on the  $Z_N$  charges have to be

chosen such that they satisfy the inequalities

$$2 q_{\psi_m} + q_{H'} \neq 0 \pmod{N}, \quad 4 q_{\psi_m} \neq 0 \pmod{N}, \quad (2a)$$

$$q_{\psi_m} + q_{\bar{\psi}_H} \neq 0 \pmod{N}, \quad 3 q_{\psi_m} + q_{\psi_H} \neq 0 \pmod{N}, \quad (2b)$$

$$q_{\psi_m} + q_{H',H} + q_{\psi_H} \neq 0 \pmod{N}, \quad q_{\psi_m} + q_{\bar{\psi}_H} + q_A \neq 0 \pmod{N}. \quad (2c)$$

The last set of constraints come from the requirement that the discrete symmetry must be a gauge symmetry i.e. it must be anomaly free. The anomaly freedom constraints are:

$$16 (N_g q_{\psi_m} + q_{\psi_H} + q_{\bar{\psi}_H}) + 10 (q_H + q_{H'}) + 45 q_A + 54 q_S = 0 \pmod{N'} \quad (3a)$$

$$2N_g q_{\psi_m} + 2 q_{\psi_H} + 2 q_{\bar{\psi}_H} + q_H + q_{H'} + 8q_A + 12q_S = 0 \pmod{N} \quad (3b)$$

$$\text{where } N' = \begin{cases} N, & \text{odd } N \\ N/2, & \text{even } N \end{cases} \quad (3c)$$

It was shown in [5] that the smallest symmetry allowing us to fulfill all criteria is  $Z_6$  for number of generation  $N_g = 3$ . A possible charge assignments is  $q_{\psi_m} = 1$ ,  $q_{\psi_H} = -2$ ,  $q_{\bar{\psi}_H} = +2$ ,  $q_H = -2$ ,  $q_{H'} = +2$ ,  $q_{45,54} = 0$  (cf. tables I (a) and (b)). This charge assignment allows for seesaw couplings and the possibility of fermion masses from couplings of type  $\psi_m \psi_m H$ . The allowed operator  $\psi_m \psi_m \bar{\psi}_H^2$  contributes to both the fermion masses as well as to the seesaw. The model also eliminates the dangerous proton decay operator  $Q Q Q L$  or operator of type  $(\psi_m)^4/M_P$ .

While the allowed set of operators provide a necessary condition for the model being phenomenologically viable, the final step where we judge whether it is acceptable first requires that we do doublet triplet splitting and see if the sub-GUT scale structure of the model can generate acceptable pattern of fermion masses or not. We address this question in the next sub-section.

### A. Phenomenological Viability of the 16-Higgs model

To analyze the phenomenological implications of the model, let us start by writing down the superpotential allowed by the discrete symmetry and  $SO(10)$  invariance:

$$\begin{aligned} W &= W_Y + W_H \\ W_Y &= h_i \psi_m \psi_m H + \frac{\lambda_{1a}}{M_{Pl}} [\psi_m \psi_m \bar{\psi}_H^2]_a + \frac{\lambda_2}{M_{Pl}} \psi_m \psi_m A H \\ W_H &= M H H' + S H H' + A H H' + \psi_H \psi_H H + \bar{\psi}_H \bar{\psi}_H H' + M_\psi \bar{\psi} \psi \end{aligned} \quad (4)$$

where  $a$  denotes the various irreducible representations in the product of two **16**'s.

TABLE I:

(a)MSSM part		(b)16-Higgs model.		(c)126-Higgs model.	
Field	quantum numbers	Field	quantum numbers	Field	quantum numbers
$\psi_m$	<b>16<sub>1</sub></b>	$\psi_H$	<b>16<sub>-2</sub></b>	$\Delta$	<b>126<sub>2</sub></b>
$H$	<b>10<sub>-2</sub></b>	$\bar{\psi}_H$	<b><math>\overline{16}_2</math></b>	$\bar{\Delta}$	<b>126<sub>-2</sub></b>
$H'$	<b>10<sub>2</sub></b>	$A$	<b>45<sub>0</sub></b>	$\Sigma$	<b>210<sub>0</sub></b>
		$S$	<b>54<sub>0</sub></b>		

First we want to find whether we can solve doublet -triplet splitting problem within this model field content. The vev s of the **54** and **45** can be assumed to have the following forms:

$$\begin{aligned}
\langle A \rangle &= \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}^{-1} \text{Diag}(a, a, a, b, b) \\
\langle S \rangle &= \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \text{Diag}(s, s, s, -\frac{3}{2}s, -\frac{3}{2}s)
\end{aligned} \tag{5}$$

It is useful to express all fields in terms of the SU(5) multiplets.

$$\begin{aligned}
16 &= 1 + \bar{5} + 10 \\
\overline{16} &= 1 + 5 + \overline{10} \\
10 &= 5 + \bar{5}
\end{aligned} \tag{6}$$

so the mass matrix in terms of SU(5) components of the fields will look like

$$(5_H, 5_{H'}, 5_{\bar{\psi}_H}) \begin{pmatrix} 0 & M + A + S & c \\ M - A + S & 0 & 0 \\ 0 & c & M_\psi \end{pmatrix} \cdot \begin{pmatrix} \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_{\psi_H} \end{pmatrix} \tag{7}$$

where  $c$  is the vev of the **16** and  $\overline{16}$  of the SO(10),  $c = \langle \psi_H \rangle = \langle \bar{\psi}_H \rangle$ . and to get expression for the mass matrix of the doublets(triplets) one has to substitute instead of  $A(S)$   $b(-\frac{3s}{2})$  for doublets and  $a(s)$  for triplets respectively. One can see that this matrix can have a zero eigenvalue only if its determinant vanishes i.e.

$$\text{Det} = (M - A + S) (c^2 - M_\psi(M + A + S)) = 0; \tag{8}$$

This equation has two solutions: taking the first one i.e.  $M - A + S = 0$ , we find for the doublet mass matrix  $M_{ud}$  in the basis  $(H, H', \psi_H)$  to be;

$$M_{ud} = \begin{pmatrix} 0 & z & c \\ 0 & 0 & 0 \\ 0 & c & M \end{pmatrix}$$

$$z = 2b = 2(M - \frac{3}{2}s). \quad (9)$$

To find the MSSM doublets in terms of the GUT submultiplets, we diagonalize  $M_{ud}$  and find its zero mode eigen-vector. The usual MSSM Higgs fields  $h_{u,d}$  will be linear combinations of  $(H, H', \psi_H)$  that correspond to the zero mode eigen-vector of the above matrix. From the following equations, we find:

$$\begin{pmatrix} 0 & z & c \\ 0 & 0 & 0 \\ 0 & c & M \end{pmatrix} \cdot \begin{pmatrix} D_{11} \\ D_{21} \\ D_{31} \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 & 0 \\ z & 0 & c \\ c & 0 & M \end{pmatrix} \cdot \begin{pmatrix} U_{11} \\ U_{21} \\ U_{31} \end{pmatrix} = 0 \quad (10)$$

It is easy to see that

$$h_u = H'_u \quad (11)$$

$$h_d = H_d$$

Since the **10**-Higgs denoted by  $H'$  does not couple to **16**-fermions, the MSSM up-Higgs doublet in this model does not couple to matter and therefore all the up quarks remain massless. The solution (i) to the determinant equation is therefore not acceptable.

Turning now to the second solution i.e. (ii)  $M + A + S = \frac{c^2}{M_\psi}$ , we get the structure of the  $M_{ud}$  mass matrix:

$$M_{ud} = \begin{pmatrix} 0 & \frac{c^2}{M} & c \\ x & 0 & 0 \\ 0 & c & M \end{pmatrix} \quad (12)$$

In this case the zero mode corresponding to the MSSM doublet  $h_d$ , can be represented by the column vector:

$$D = \begin{pmatrix} 0 \\ -\frac{M}{c\sqrt{1+\frac{M^2}{c^2}}} \\ \frac{1}{\sqrt{1+\frac{M^2}{c^2}}} \end{pmatrix}$$

$$U = \begin{pmatrix} -\frac{M}{c\sqrt{1+\frac{M^2}{c^2}}} \\ 0 \\ \frac{1}{\sqrt{1+\frac{M^2}{c^2}}} \end{pmatrix} \quad (13)$$

The MSSM doublet  $h_d$  in this case does not couple to quarks and charged leptons and leave those fields massless. Again this is not acceptable. Taking these two cases together we conclude that in the minimal gauge discrete symmetric  $Z_6$  model, the doublet triplet splitting and nontrivial fermion masses cannot happen simultaneously and the model is therefore not phenomenologically viable. We wish to emphasize again that in drawing this conclusion, we have also considered higher dimensional operators that could contribute fermion masses. It turns out that in our, case operators such as  $(\psi_m)^2(\psi_H)^2$ ,  $(\psi_m)^2 H' A$  which can lead to the bottom quark mass are not allowed due to the  $Z_6$  charge assignments.

### B. Phenomenological viability of the $SO(10) \times Z_6$ 126 model

This class of models typically have the Higgs multiplets of type **10 126,210** (and **120**) fields to explain fermion masses including neutrino masses and mixings. These models do not have R-parity breaking terms even after GUT symmetry breaking. The  $Z_6$  charge assignments that makes the model proton-decay-safe while keeping necessary terms for possible fermion masse are given in Table I (c).

The Higgs superpotential in this case looks like

$$W = M_H H H' + M_\Delta \Delta \bar{\Delta} + \lambda_{\Delta \Sigma H} \Delta \Sigma H + M_\Sigma \Sigma^2 + \lambda_{\Sigma^3} \Sigma^3 + \lambda_{\bar{\Delta} \Sigma H'} \bar{\Delta} \Sigma H' + \lambda_{\bar{\Delta} \Sigma \Delta} \bar{\Delta} \Sigma \Delta. \quad (14)$$

One might think that this can lead to a realistic model for fermion masses. However as in the previous sub-section, we must analyze the doublet-triplet splitting in order to study the fermion



masses. we will see that in this case too there is a conflict between the doublet-triplet splitting and fermion masses.

To see this, we write down the mass matrix for MSSM doublets contained in various GUT Higgs multiplets in the theory: (see [7] for the exact Clebsch -Gordon coefficients)

$$(H_u, H'_u, \Delta_u, \bar{\Delta}_u, \Sigma_u) \times M_{ud} \times \begin{pmatrix} H_d \\ H'_d \\ \bar{\Delta}_d \\ \Delta_d \\ \Sigma_d \end{pmatrix};$$

$$M_{ud} = \begin{pmatrix} 0 & M_H & 0 & -\frac{\lambda H \Delta \Sigma}{\sqrt{10}}(\Phi_2 + \frac{\Phi_3}{\sqrt{2}}) & -\frac{\lambda H \Delta \Sigma}{\sqrt{5}}v \\ M_H & 0 & \frac{\lambda H' \bar{\Delta} \Sigma}{\sqrt{10}}(\Phi_2 - \frac{\Phi_3}{\sqrt{2}}) & 0 & 0 \\ \frac{\lambda H \Delta \Sigma}{\sqrt{10}}(\Phi_2 - \frac{\Phi_3}{\sqrt{2}}) & 0 & M_\Delta + \frac{\lambda \Sigma \Delta \bar{\Delta}}{15\sqrt{2}}(\Phi_2 - \frac{\Phi_3}{\sqrt{2}}) & 0 & 0 \\ 0 & -\frac{\lambda H' \bar{\Delta} \Sigma}{\sqrt{10}}(\Phi_2 + \frac{\Phi_3}{\sqrt{2}}) & 0 & M_\Delta + \frac{\lambda \Sigma \Delta \bar{\Delta}}{15\sqrt{2}}(\Phi_2 + \frac{\Phi_3}{\sqrt{2}}) & \frac{\lambda \bar{\Delta} \Delta \Sigma}{10}v \\ 0 & -\frac{\lambda H' \bar{\Delta} \Sigma}{\sqrt{5}}\bar{v} & 0 & \frac{\lambda \bar{\Delta} \Delta \Sigma}{10}\bar{v} & M_\Sigma + \frac{\lambda \Sigma^3}{\sqrt{2}}(\Phi_2 + \frac{\Phi_3}{\sqrt{2}}) \end{pmatrix} \quad (15)$$

Where  $\Phi_1, \Phi_2, \Phi_3$  are the vevs of  $\Sigma$  in different directions and  $v, \bar{v}$  vevs of  $\Delta, \bar{\Delta}$  fields respectively.

For simplicity of analysis, we rewrite this matrix in the symbolic form as follows:

$$M_{ud} = \begin{pmatrix} 0 & m & 0 & \frac{da}{b} & c \\ m & 0 & b & 0 & 0 \\ a & 0 & M & 0 & 0 \\ 0 & d & 0 & M_2 & A \\ 0 & c_1 & 0 & A & B \end{pmatrix} \quad (16)$$

We want to have one massless state so we require that

$$\text{Det} M_{ud} = 0. \quad (17)$$

This requires that one of the following two conditions be satisfied  $a = \frac{mM}{b}$  or  $a = \frac{Abc_1d - A^2bm - bcc_1M_2 + BbmM_2}{d(Bd - Ac_1)}$ . Let us now investigate the first one (i)  $a = \frac{mM}{b}$  which leads to the following zero mass eigenstate:

$$U = \begin{pmatrix} 0 \\ -\frac{M}{b\sqrt{1 + \frac{M^2}{b^2}}} \\ \frac{1}{\sqrt{1 + \frac{M^2}{b^2}}} \\ 0 \\ 0 \end{pmatrix} \quad (18)$$

$$D = \begin{pmatrix} -\frac{b}{m\sqrt{1+\frac{b^2}{m^2}}} \\ 0 \\ \frac{1}{\sqrt{1+\frac{b^2}{m^2}}} \\ 0 \\ 0 \end{pmatrix} \quad (19)$$

In this case we see that the up-MSSM Higgs doublet does not couple to matter fermions.

Turning now to the case (ii) where we have  $a = \frac{Abc_1d - A^2bm - bcc_1M_2 + BbmM_2}{d(Bd - Ac_1)}$ , we get for the same eigenstates:

$$U = \begin{pmatrix} * \\ 0 \\ 0 \\ * \\ * \end{pmatrix} \quad (20)$$

$$D = \begin{pmatrix} 0 \\ * \\ 0 \\ * \\ * \end{pmatrix} \quad (21)$$

where we have written only the zero entries in the columns. The \*'s represent non-zero entries whose detailed form is irrelevant for our discussion. It is clear that in both cases the doublet-triplet splitting is incompatible with giving masses to the fermions; in the case (i) to up quarks and in case (ii) to the down quarks.

So neither the minimal **16** nor **126** models when made proton decay safe can lead to viable fermion masses along with doublet triplet splitting. We therefore have to extend the Higgs sector to get a realistic model. In the next section, we give one such example and analyze its flavor phenomenology.

### III. EXTENDED 16-HIGGS MODEL

We now extend the **16** -Higgs model by adding extra Higgs multiplets in such a way that anomaly freedom as well as proton decay constraints are satisfied and yet the model can lead to

viable phenomenology. The simplest possibility appears to be to extend our **16** model by adding one additional **10** -  $H_3$  and two **45** -  $A_2, A_3$  fields under  $SO(10)$ , where  $H_3$  has zero charge under  $Z_6$ , and  $A_{2,3}$  charges are -2 and 2 respectively (see table below).  $Z_6$  charges are easily seen not to

TABLE II:

Field	quantum numbers
$A_2$	<b>45</b> <sub>-2</sub>
$A_3$	<b>45</b> <sub>2</sub>
$H_3$	<b>10</b> <sub>0</sub>

ruin our anomaly cancellation conditions. We also have redefined our  $H', H$  fields as  $H_{1,2}$  and  $A$  as  $A_1$  for the simplicity of notation . Now the superpotential is given by:

$$W = MH_1H_2 + SH_1H_2 + A_1H_1H_2 + \psi_H\psi_HH_1 + \bar{\psi}_H\bar{\psi}_HH_2 + M_\psi\bar{\psi}_H\psi_H + M_3H_3^2 + H_2H_3A_2 + H_1H_3A_3 + \frac{H_3A_3\bar{\psi}_H\bar{\psi}_H}{M_{Pl}} + \frac{H_3A_2\psi_H\psi_H}{M_{Pl}} \quad (22)$$

Our model allows an operator of the form  $\frac{\psi_m^4 A_3}{M_{Pl}}$  where substituting the vev of the field  $A_3$  we get a proton decay operator with effective  $\lambda \simeq \frac{M_U}{M_{Pl}} \ll 1$  but not suppressed enough to be acceptable. However this problem disappears if the vev  $\langle A_3 \rangle = 0$ . We will see below that there is an allowed vacuum, where indeed this is possible.

To study the doublet triplet splitting in this model, note that the mass matrix for the **5** and  $\bar{\mathbf{5}}$  of  $SU(5)$  is given by

$$\begin{pmatrix} 0 & M + A_1 + S & 0 & c \\ M - A_1 + S & 0 & A_2 & 0 \\ 0 & -A_2 & M_3 & 0 \\ 0 & c & \delta & M_\psi \end{pmatrix}_{ud} \quad (23)$$

Where  $\delta$  in the (43) element of the matrix comes from  $\frac{H_3A_2\psi_H\psi_H}{M_{Pl}}$  coupling. As before, we want the determinant of this matrix to vanish. This leads to the following constraints

Case (i)

$$M - A_1 + S = 0 \quad (24)$$

Case(ii):

$$M + A_1 + S = \frac{c^2 M_3 + c A_2 \delta}{M_3 M_\psi} \quad (25)$$

Here we consider only the simpler of the two cases above i.e. case (i) to illustrate that our proposal leads to a realistic model. In the first case  $M - A_1 + S = 0$ ,  $D = (1, 0, 0, 0)$  (implying that the  $h_d$  has non-zero component in the multiplet  $H_1$ ) in the same way as was in the minimal model, but now due to the presence of  $A_2$  field all the  $U_1$  is nonvanishing, so that the "up" quarks will get masses from the  $\psi_m^2 H_1$  operator. In the next sections we will discuss the detailed fit to fermion masses for this extended **16** model.

As we can see extended **16** model can solve doublet-triplet splitting problem as well as provide masses for all fermions, but now we have to check whether higgsino mediated proton decay operators are allowed. Even though quarks and leptons couple only to the  $H_1$  field and there is no mass term  $\propto H_1 H_1$ , mixing between  $H_1, H_2, H_3, \psi_H$  fields can lead to the nonvanishing diagrams with higgsino exchange. The contribution of these diagrams will vanish if only the  $(H_1 H_1)$  element of the inverse mass matrix (23) for the heavy triplets vanishes, thus the triplet part of the  $A_2$  should be zero. We will see in the next section that the requirement of the  $\langle A_3 \rangle = 0$  combined with F flatness condition will lead to this condition.

#### IV. SUPERSYMMETRY DOWN TO THE WEAK SCALE

First we want to find out whether there is a minimum of the potential that can correspond to the solution we are interested in i.e. having supersymmetry survive down to the weak scale. The Higgs part of the superpotential is:

$$W_H = M_\psi \psi_H \bar{\psi}_H + m_1 A_1^2 + m_2 A_2 A_3 + m_s S^2 + \lambda_1 A_1^2 S + \lambda_2 S^3 + \lambda_3 A_2 A_3 S + \lambda_4 \bar{\psi}_H \psi_H A_1 + \lambda_5 A_1 A_2 A_3 \quad (26)$$

+nonrenormalizable terms

The vev of the **45**, **54** and **16** fields will in general have the following form:

$$\begin{aligned} \langle A_i \rangle &= \begin{pmatrix} -1 & 1 \end{pmatrix} (a_i, a_i, a_i, b_i, b_i); \\ \langle S \rangle &= \begin{pmatrix} 1 & 1 \end{pmatrix} (s, s, s, -\frac{3}{2}s, -\frac{3}{2}s); \\ \langle \bar{\psi}_H \rangle &= \langle \psi_H \rangle = c \end{aligned} \quad (27)$$

So we can rewrite the superpotential in terms of the vev of these fields, using the further identities:

$$\begin{aligned}
M_\psi \psi_H \bar{\psi}_H &= M_\psi c^2 \\
m_1 A_1^2 &= -2(3a_1^2 + 2b_1^2)m_1 \\
m_2 A_3 A_2 &= -2(3a_2 a_3 + 2b_2 b_3)m_2 \\
\lambda_1 A_1^2 S &= (-6a_1^2 s + 6b_1^2 s)\lambda_1 \\
\lambda_3 A_2 A_3 S &= (-6a_2 a_3 s + 6b_2 b_3 s)\lambda_3 s \\
\lambda_2 S^3 &= -\frac{15}{2}\lambda_2 s^3 \\
m_s S^2 &= 15m_s s^2
\end{aligned} \tag{28}$$

The condition of the vanishing F terms leads to the following constraints,

$$\begin{aligned}
\frac{\partial W}{\partial a_1} &= -12m_1 a_1 - 12\lambda_1 a_1 s + 3\lambda_4 c^2 = 0 \\
\frac{\partial W}{\partial b_1} &= -8m_1 b_1 + 12\lambda_1 b_1 s + 2\lambda_4 c^2 = 0 \\
\frac{\partial W}{\partial a_2} &= -6m_2 a_3 - 6\lambda_3 a_3 s = 0 \\
\frac{\partial W}{\partial b_2} &= -4m_2 b_3 + 6\lambda_3 b_3 s = 0 \\
\frac{\partial W}{\partial a_3} &= -6m_2 a_2 - 6\lambda_3 a_2 s = 0 \\
\frac{\partial W}{\partial b_3} &= -4m_2 b_2 + 6\lambda_3 b_2 s = 0 \\
\frac{\partial W}{\partial s} &= 30m_s s + \lambda_1(6b_1^2 - 6a_1^2) + \lambda_3(-6a_2 a_3 + 6b_2 b_3) - \frac{45}{2}\lambda_2 s^2 = 0 \\
\frac{\partial W}{\partial c} &= 2cM_\psi + 2c\lambda_4(3a_1 + 2b_1) = 0
\end{aligned} \tag{29}$$

we are interested in whether there exist a solution with  $a_3 = b_3 = 0$  and  $b_2 \neq 0$  these constraints lead to the following restrictions on the vevs

$$\begin{aligned}
a_2 = 0, \quad s &= \frac{2m_2}{3\lambda_3} \\
(3a_1 + 2b_1)\lambda_4 &= -M_\psi, \quad s = \frac{2m_1(b_1 - a_1)}{\lambda_1(2a_1 + 3b_1)}
\end{aligned} \tag{30}$$

required to suppress higgsino exchange diagrams. Now we will present the other massless components of the higgs fields that provide the breaking of the  $SO(10) \rightarrow SU(3) \times SU(2) \times U(1)$ .

We will identify them by their charges under  $SU(3) \times SU(2) \times U(1)$

1) (3,1,2/3) fields ( $A_1, A_2, A_3, \psi_H$ );

$$\begin{pmatrix} -4m_1 - 4\lambda_1 s & 0 & 0 & -2\lambda_4 c \\ 0 & 0 & -2m_2 - 2\lambda_3 s + 2ia_1\lambda_5 & 0 \\ 0 & -2m_2 - 2\lambda_3 s - 2ia_1\lambda_5 & 0 & 0 \\ -2\lambda_4 c & 0 & 0 & \lambda_4(2b_1 - a_1) + M_\psi \end{pmatrix} \quad (31)$$

2) (3,2,-5/6) fields  $(A_1, A_2, A_3, S)$  ;

$$\begin{pmatrix} -4m_1 + \lambda_1 s & 0 & -ib_2\lambda_5 & 2i(a_1 + b_1)\lambda_1 \\ 0 & 0 & -2m_2 - \lambda_3 \frac{s}{2} + i(a_1 - b_1)\lambda_5 & 0 \\ ib_2\lambda_5 & -2m_2 - \lambda_3 \frac{s}{2} - i(a_1 - b_1)\lambda_5 & 0 & i\lambda_3 b_2 \\ -2i(a_1 + b_1)\lambda_1 & 0 & -i\lambda_3 b_2 & 4m_s - 3\lambda_2 s \end{pmatrix} \quad (32)$$

3) (3,2,1/6) fields  $(A_1, A_2, A_3, S, \psi_H)$ ;

$$\begin{pmatrix} -4m_1 + \lambda_1 s & 0 & -ib_2\lambda_5 & 2i(a_1 - b_1)\lambda_1 & -2\lambda_4 c \\ 0 & 0 & -2m_2 + \lambda_3 \frac{s}{2} + i(a_1 + b_1)\lambda_5 & 0 & 0 \\ ib_2\lambda_5 & -2m_2 + \lambda_3 \frac{s}{2} - i(a_1 + b_1)\lambda_5 & 0 & -ib_2\lambda_3 & 0 \\ -2i(a_1 - b_1)\lambda_1 & 0 & ib_2\lambda_3 & 4m_s - 3\lambda_2 s & 0 \\ -2\lambda_4 c & 0 & 0 & 0 & M_\psi + \lambda_4 a_1 \end{pmatrix} \quad (33)$$

4) (1,1,1) fields  $(A_1, A_2, A_3, \psi_H)$ ;

$$\begin{pmatrix} -4m_1 + 6\lambda_1 s & 0 & -2i\lambda_5 b_2 & 2\lambda_4 c \\ 0 & 0 & -2m_2 + 3\lambda_3 s + 2ib_1\lambda_5 & 0 \\ 2i\lambda_5 b_2 & -2m_2 + 3\lambda_3 s + 2ib_1\lambda_5 & 0 & 0 \\ 2\lambda_4 c & 0 & 0 & \lambda_4(3a_1 - 2b_1) + M_\psi \end{pmatrix} \quad (34)$$

from the equations (29-30) one can see that each of these matrices will have one massless eigenstate. So we have total 32 massless goldstone bosons. One more goldstone boson needed to break  $SO(10)$  down to  $SU(2) \times SU(3) \times U(1)$  comes from the phase of the  $\psi_H, \bar{\psi}_H$  fields

## V. FERMION MASSES

The following couplings allowed by  $Z_6 \times SO(10)$  symmetries will lead to fermion masses after symmetry breaking.

$$W = h^{10} \psi_m \psi_m H_1 + f^{10} \frac{\psi_m^2 \bar{\psi}_H^2}{M_{Pl}} + f^{126} \frac{\psi_m^2 \bar{\psi}_H^2}{5! M_{Pl}} + k^{120} \frac{\psi_m^2 A H_1}{3! M_{Pl}} + g^{120} \frac{\psi_m^2 A^2 H_1}{3! M_{Pl}^2} + g^{126} \frac{\psi_m^2 A^2 H_1}{5! M_{Pl}^2} \quad (35)$$

Where  $h^{10}, f^{10}, f^{126}, g^{126}$  are symmetric  $3 \times 3$  and  $k^{120}, g^{120}$  antisymmetric matrices. The upper index of  $h^{10}, f^{126}, \dots, \mathbf{10}, \mathbf{120}, \mathbf{126}$  shows the SO(10) structure of the fermion couplings.

$$\begin{aligned}
M_u &= v_u \left[ \alpha \left( h^{10} - k^{120} \frac{a-b}{M_{Pl}} + g^{120} \frac{b(a-b)}{M_{pl}^2} + g^{126} \frac{ab+a^2}{M_{Pl}^2} \right) + \xi \left( f^{10} \frac{4c}{M_{Pl}} - f^{126} \frac{8c}{M_{Pl}} \right) \right] \\
M_\nu^D &= v_u \left[ \alpha \left( h^{10} + k^{120} \frac{3a+b}{M_{Pl}} - g^{120} \frac{b(3a+b)}{M_{pl}^2} - g^{126} \frac{3(ab+a^2)}{M_{pl}^2} \right) + \xi \left( f^{10} \frac{4c}{M_{Pl}} + f^{126} \frac{24c}{M_{Pl}} \right) \right] \\
M_d &= v_d \gamma \left[ h^{10} - k^{120} \frac{a+b}{M_{Pl}} - g^{120} \frac{b(a+b)}{M_{pl}^2} + g^{126} \frac{-ab+a^2}{M_{pl}^2} \right] \\
M_e &= v_d \gamma \left[ h^{10} + k^{120} \frac{3a-b}{M_{Pl}} + g^{120} \frac{b(3a-b)}{M_{pl}^2} - g^{126} \frac{3(-ab+a^2)}{M_{pl}^2} \right] \\
M_\nu^M &= 16 f^{126} \frac{(\xi v_u)^2}{M_{pl}} - (M_\nu^D)^T (16 f^{126} \frac{c^2}{M_{Pl}})^{-1} M_\nu^D
\end{aligned} \tag{36}$$

Where the vev of the fields  $H^1$  and  $\bar{\psi}_H$  are related to the vev of the MSSM doublets  $h_u$  and  $h_d$  in the following way

$$\begin{aligned}
\langle H_{1u} \rangle &= \alpha \langle h_u \rangle \\
\langle H_{1d} \rangle &= \gamma \langle h_d \rangle \\
\langle \bar{\psi}_{Hu} \rangle &= \xi \langle h_u \rangle
\end{aligned} \tag{37}$$

Our claim is that these Yukawa coupling structure is rich enough to fit all the fermion masses. We give below an example of a scenario where correct fermion masses can arise.

We take the case where all antisymmetric couplings vanish and that down quarks and leptons are brought to the diagonal basis at the same time. Thus  $g^{126}$  and  $h^{10}$  are diagonal, and all the mixing in the quark and lepton sector arise from the couplings  $f^{10}$  and  $f^{126}$ . We now show that even under such limiting assumptions we can fit all the fermion masses. We know the quark masses at the GUT scale thus we can find corresponding  $h^{10}, g^{126}$ , but on the other hand in the case when all the quark mass matrices are symmetric the mass matrix for the up quarks is equal to

$$M_u = (CKM)^T \cdot M_u^{diag} \cdot CKM; \tag{38}$$

this leads to the constraint on the linear combination of  $f^{10}$  and  $f^{126}$ . On the other hand we can find  $f_{126}$  from the known neutrino masses and mixing angles [9], so this fixes  $f^{10}$  and  $f^{126}$ .

So here is the fit for  $a, b, c, \alpha, \xi$  that leads to the good quark and lepton mixing. We set  $\tan(\beta) = 55$ , the masses of the quarks and leptons and the vev's  $v_u, v_d$  at the GUT could be found in [8].

$$\begin{aligned}
\alpha &= 0.8 \\
\xi &= -0.47 \\
\gamma &= 1 \\
a &= 0.028 M_{Pl} \\
b &= -0.014 M_{Pl} \\
c &= -0.024 M_{Pl}
\end{aligned} \tag{39}$$

$$\begin{aligned}
h_{10} &= \text{Diag}(0.000574402, 0.0208157, 0.792778) \\
g_{126} &= \text{Diag}(0.114692, -4.43859, 0.190027) \\
f_{10} &= \begin{pmatrix} -0.00858937 - 0.000536445i & -0.0110172 + 0.00138491i & 0.094032 - 0.039795i \\ -0.0110172 + 0.00138491i & -0.286505 + 0.000708062i & -0.508733 - 0.00956549i \\ 0.094032 - 0.039795i & -0.508733 - 0.00956549i & -0.134644 - 0.0176603i \end{pmatrix} \\
f_{126} &= \begin{pmatrix} -5.509 \times 10^{-6} + 5.992 \times 10^{-6}i & 0.00003070 - 0.00001463i & -0.001095 + 0.0004482i \\ 0.00003070 - 0.00001463i & -0.00014401 + 9.1768 \times 10^{-6}i & 0.004929 - 0.00003607i \\ -0.001095 + 0.0004482i & 0.004929 - 0.00003607i & -0.1718 - 0.008830i \end{pmatrix}
\end{aligned}$$

these Yukawa couplings lead to the good mass matrices for the up, down quarks, charged leptons and neutrinos. Note that there are no dimension five operators that contribute to down quark mass



matrix due to the discrete symmetry of the model.

$$\begin{aligned}
M_d &= \text{Diag}(1.46323, 32.2949, 1638.17)\text{MeV} \\
M_e &= \text{Diag}(0.35668, 75., 1636.)\text{MeV} \\
M_u &= \text{Diag}(0.757795, 208.466, 87029.8)\text{MeV} \\
M_u &= \begin{pmatrix} 14.74 - 3.341i & -67.49 + 8.615i & 586.2 - 247.9i \\ -67.49 + 8.615i & 312.8 + 4.202i & -3159. - 57.83i \\ 586.2 - 247.9i & -3159. - 57.83i & 86910. \end{pmatrix} \text{MeV} \\
v_u &= 135016.\text{MeV}, \quad v_d = 2065.81\text{MeV} \\
M_\nu^2 &= \text{Diag}(9.577 * 10^{-7}, 0.00007594, 0.002697)\text{eV}^2 \\
U_{ei} &= \begin{pmatrix} -0.7995 & 0.59481 & -0.083258 \\ -0.35036 - 0.14031i & -0.51466 - 0.090935i & -0.31226 + 0.69779i \\ -0.46449 - 0.050427i & -0.59179 - 0.15112i & 0.2327 - 0.5954i \end{pmatrix} \\
V_{CKM} &= \begin{pmatrix} 0.973841 & 0.227198 & 0.00169092 - 0.00292876i \\ -0.227079 - 0.000134603i & 0.97298 - 0.000031403i & 0.0369876 \\ 0.00675874 - 0.00284968i & -0.0364044 - 0.000664834i & 0.99912 \end{pmatrix} \quad (41)
\end{aligned}$$

The FIG.1 shows the distribution for the values of  $\sin^2\theta_{13}$  in our model for  $2\sigma$  values of neutrino masses and mixings. As is clear from this figure the model has a slight preference towards the region of small  $\theta_{13}$ . The distribution for the other parameters of the neutrino mass matrices appear to be spread uniformly over the allowed regions.

## VI. CONSTRAINTS FROM THE LEPTON FLAVOR VIOLATION

In this section, we discuss the predictions of this model for lepton flavor violation. As is well known[10, 11], even if the slepton mass matrices are diagonal at the GUT scale the RGE running down to the scale of the righthanded neutrino will lead to the mixing in the slepton sector, which via one loop diagrams leads to lepton violation. We will assume mSUGRA boundary condition for scalar partner masses and use the renormalization group equations to run them down to the seesaw scale when the right handed neutrinos decouple.

We will work in the basis with diagonal righthanded majorana neutrino matrix, then the slepton

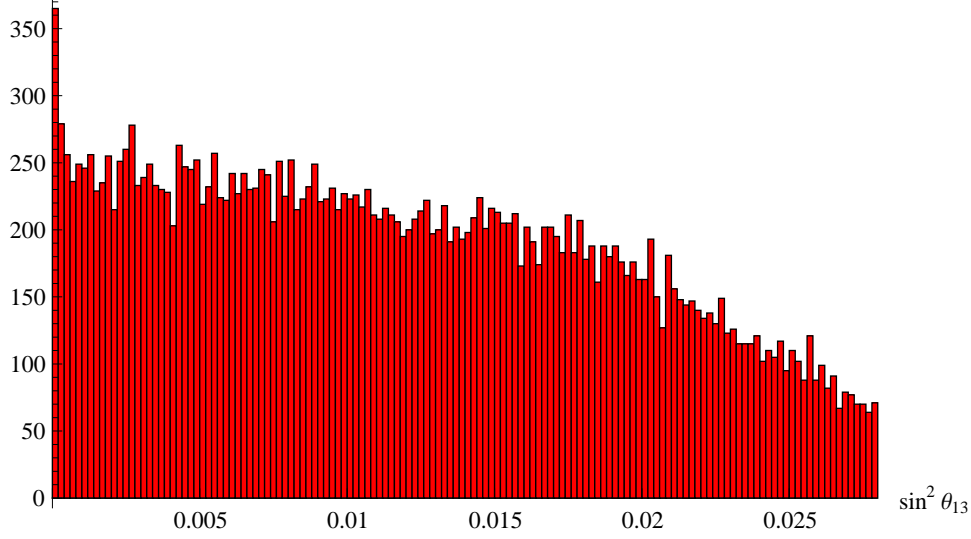


FIG. 1: Distribution plot for the values of  $\sin^2 \theta_{13}$

mixing will be approximately equal to

$$(\delta_{ij}^l)_{LL} = -\frac{3m_0^2 + A_0^2}{8\pi^2 m_0^2} \sum_{k=1}^3 (Y_\nu)_{ik} (Y_\nu^*)_{jk} \ln\left(\frac{M_{GUT}}{M_{R_k}}\right) \quad (42)$$

where  $Y_\nu$  are the Yukawa couplings of the Dirac neutrino. These Yukawa couplings appear to be of roughly

$$Y_\nu \sim \begin{pmatrix} 10^{-5} & 5 \cdot 10^{-4} & 5 \cdot 10^{-3} \\ 10^{-5} & 5 \cdot 10^{-3} & 2 \cdot 10^{-2} \\ 10^{-5} & 3 \cdot 10^{-3} & 0.3 \end{pmatrix} \quad (43)$$

Here  $Y_\nu$  is a linear combination of the Yukawa couplings  $h_{10}$ ,  $f_{10}$  and  $f_{126}$  of the previous section. The slepton mixing leads to the lepton flavor violating processes  $l_i \rightarrow l_j \gamma$  with the amplitude equal to

$$iM = em_{li} \epsilon^{\lambda} \bar{l}_j (iq^\mu \sigma_{\lambda\mu} (A_L P_L + A_R P_R)) l_i \quad (44)$$

Where the  $q$  is the momentum of the photon and  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ , the exact expression for the  $A_{L,R}$  can be found in [11]. The branching ratio for this processes will be equal to

$$\frac{BR(l_i \rightarrow l_j \gamma)}{BR(l_i \rightarrow l_j \nu_i \bar{\nu}_j)} = \frac{48\pi^3 \alpha}{G_F^2} (|A_L^{ij}|^2) \quad (45)$$

$$A_l^{ij} \propto \frac{\alpha_2}{4\pi} \frac{(\delta_{ij}^l)_{LL}}{m_0^2}$$

$BR(\mu \rightarrow e\gamma)$	$1.2 \cdot 10^{-11}$
$BR(\tau \rightarrow \mu\gamma)$	$6.8 \cdot 10^{-8}$

The present bounds on this processes are[12]

We will carry out our calculations for the branching ratio in the mSUGRA scenario, where there are only four parameters that will fix the low energy values of the slepton masses  $M_{1/2}, m_0, A_0, \tan\beta, \text{sign}(\mu)$  but our fit for the fermion masses was carried out for the  $\tan\beta = 55$  so we will stay with this value. In the FIG.2 one can see dependence of the branching ratios on the

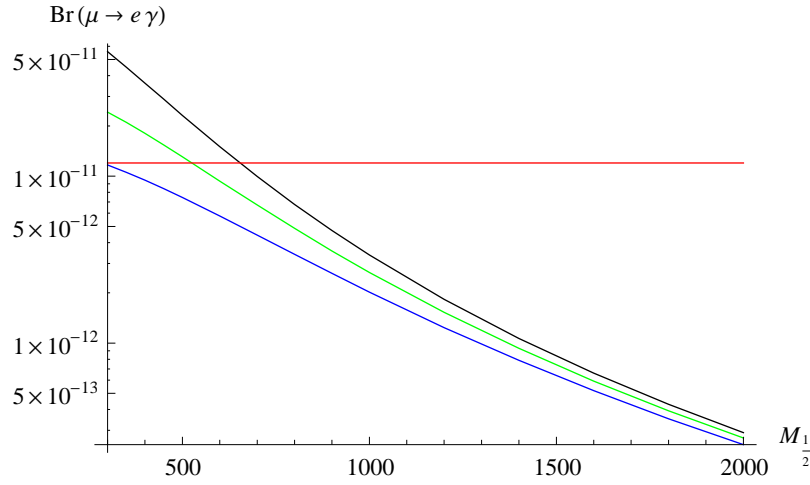


FIG. 2: Branching ratios  $BR(\mu \rightarrow e\gamma)$  for different values of  $m_0$  (black 300 GeV, green 400 GeV, blue 500 GeV),  $A_0 = 0, \tan\beta = 55, \text{sign}(\mu)=1$

$M_{1/2}$  for the fixed values of  $m_0, \tan\beta, A_0, \text{sign}(\mu)$ . We note that branching ratio for  $\mu \rightarrow e + \gamma$  for almost the entire parameter range of our model is above  $10^{-13}$  a value which is in the accessible range of the ongoing MEG experiment[12].

## VII. COMMENTS

We add a few comments on the model described before closing:

(i) In this model, the leading order proton decay operator is  $\frac{\psi_m^4 A_2^2}{M_{Pl}^3}$ . After GUT symmetry breaking this leads to the effective strength  $\lambda \sim \frac{M_U^2}{M_{Pl}^2}$ . Naively this is of order  $2 \times 10^{-5}$ , bigger than the present upper limit but is a considerable improvement in the naturalness. It could also be that the GUT vev could arise mainly from  $A_1$  with  $\langle A_2 \rangle$  being an order of magnitude smaller. This would

then give the desired suppression to proton decay. In that case this will be the dominant graph for proton decay. Note that there are no Higgsino mediated diagrams for proton decay in this model. In addition, there is the gauge exchange diagram, present in all SO(10) GUT models.

(ii) The  $\mu \rightarrow e + \gamma$  appears to be the only other low energy test of the model which is similar to such models.

(iii) For the choice of parameters used in fermion mass fitting the neutrino mixing angles and mass differences could have any values in the allowed region.

## VIII. CONCLUSION

In conclusion, we have presented the minimal SO(10) **16**-Higgs model for fermion masses where the problem of extreme fine tuning of higher dimensional Planck scale induced proton decay operators has been considerably ameliorated by the presence of discrete symmetries so that in the end, we only need to tune down the coupling only by a factor of  $10^{-2}$ . In this sense it is a more natural model. We exhibited a fit to all fermion masses and mixings including neutrinos in this model to show that it can indeed be a realistic description of nature.

This work is supported by the National Science Foundation Grant No. PHY-0652363. This work was presented at the GUT2007 workshop at Ritsumeikan University in Japan by R. N. M. and we like to thank the participants at this workshop and M. Ratz for comments.

- 
- [1] T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B **115**, 26 (1982); C. S. Aulakh and R. N. Mohapatra, Phys. Rev. D **28**, 217 (1983); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993); D. G. Lee and R. N. Mohapatra, Phys. Rev. D **51**, 1353 (1995); M. C. Chen and K. T. Mahanthappa, Phys. Rev. D **62**, 113007 (2000); T. Fukuyama and N. Okada, hep-ph/0206118; B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003) hep-ph/0210207. H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B **570**, 215 (2003) [arXiv:hep-ph/0303055]; T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, JHEP **0409**, 052 (2004); Eur.Phys.J.C42:191-203,2005; B. Dutta, Y. Mimura and R. N. Mohapatra, hep-ph/0406262, Phys. Lett. B **603**, 35 (2004); Phys. Rev. Lett. **94**, 091804 (2005); Phys. Rev. D **72**, 075009 (2005); B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Lett. B **634**, 272 (2006); C. S. Aulakh and S. K. Garg, hep-ph/0512224; K. S. Babu and

- C. Macesanu, Phys. Rev. **D 72**, 115003 (2005); S. Bertolini, M. Malinsky and T. Schwetz, Phys. Rev. **D 73**, 115012 (2006); W. Grimus and H. Kuhbock, hep-ph/0612132; arXiv:0710.1585 [hep-ph].
- [2] S. M. Barr and S. Raby, Phys. Rev. Lett. **79**, 4748 (1997) C. Albright, K. S. Babu and S. Barr, Phys. Rev. Lett. **81**, 1167 (1998); C. Albright and S. Barr, Phys. Rev. **D 58**, 013002 (1998); K. S. Babu, J. C. Pati and F. Wilczek, hep-ph/9812538; S. Raby, Phys. Rev. **D 65**, 115004 (2002); R. Dermisek and S. Raby, Phys. Lett. B **622**, 327 (2005); X. Ji, Y. Li and R. N. Mohapatra, Phys. Lett. **B 633**, 755 (2006); R. Dermisek, M. Harada and S. Raby, Phys. Rev. **D 74**, 035011 (2006); S. Morisi, M. Picariello and E. Torrente-Lujan, Phys. Rev. **D 75**, 075015 (2007)
- [3] I. Hinchliffe and T. Kaeding, Phys. Rev. **D47** (1993), 279; H. K. Dreiner, C. Luhn, and M. Thormeier, Phys. Rev. **D73** (2006), 075007, hep-ph/0512163; L. E. Ibáñez and G. G. Ross, Phys. Lett. **B260** (1991), 291; T. Banks and M. Dine, Phys. Rev. **D45** (1992), 1424, hep-th/9109045; K. S. Babu, I. Gogoladze and K. Wang, Phys. Lett. **B570** (2003), 32, hep-ph/0306003; K. Kurosawa, N. Maru and T. Yanagida, Phys. Lett. **B512** (2001), 203, arXiv:hep-ph/0105136; Y. Kajiyama, E. Itou and J. Kubo, Nucl. Phys. **B743** (2006), 74, hep-ph/0511268.
- [4] For other approaches to suppressing proton decay, see P. Nath and R. Sayed, Phys. Rev. **D 77** (2008) 015015; H. Dreiner and M. Thormeier, Phys. Rev. **D 69** (2004) 053002; J. Sayre, S. Wiesenfeldt and S. Willenbrock, Phys. Rev. **D 75** (2007) 037702; for a review of proton decay, see Pran Nath and Pavel Fileviez Perez, Phys.Rept. **441** (2007)191-317.
- [5] R. N. Mohapatra and M. Ratz, arXiv:0707.4070 [hep-ph], Phys. Rev. **D 76**, 095003 (2007).
- [6] K. S. Babu and S. M. Barr, Phys. Rev. **D 48**, 5354 (1993); Z. Chacko and R. N. Mohapatra, Phys. Rev. **D 59**, 011702 (1999); R. Dermisek, A. Mafi and S. Raby, Phys. Rev. **D 63**, 035001 (2001).
- [7] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, J. Math. Phys. **46**, 033505 (2005) [arXiv:hep-ph/0405300].
- [8] C. R. Das and M. K. Parida, Eur. Phys. J. C **20**, 121 (2001) [arXiv:hep-ph/0010004].
- [9] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **6**, 122 (2004) [arXiv:hep-ph/0405172].
- [10] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986). J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357**, 579 (1995) [arXiv:hep-ph/9501407]; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. **D 53**, 2442 (1996) [arXiv:hep-ph/9510309].
- [11] A. Masiero, S. K. Vempati and O. Vives, arXiv:0711.2903 [hep-ph]; M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S. K. Vempati and O. Vives, Nucl. Phys. B **783**, 112 (2007) [arXiv:hep-

ph/0702144].

[12] C. Bemporado et al. MEG Collaboration; PSI-R-99-05.